# Time-Weighted Rate of Return - a Special Case of the Money-Weighted Rate of Return 

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# Time-Weighted Rate of Return - a Special Case of the Money-Weighted Rate of Return 

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## INTRODUCTION

Practitioners and academics have been discussing for years the use as well as the advantages and disadvantages of the two common concepts of return measurement in asset management: the Time-Weighted Rate of Return (TWR) and the Money-Weighted Rate of Return (MWR). These discussions were sometimes very intense and even emotional, suggesting that they are two fundamentally different concepts of return measurement. This article explores this assumption and the question of whether the two concepts of return measurement are fundamentally different. If this is not the case, then there would be a superordinate concept of return measurement and the TWR and the MWR as well as their methodological variations would be "only" exceptional cases of a generic concept of return measurement.

## MWR AS THE ORIGINAL CONCEPT OF RETURN MEASUREMENT

In the following, the MWR will be referred to as the original concept of return measurement, since it is the concept that has been used for a very long time in economics to measure the profitability of investments. The original concept puts the profit and loss in relation to the capital invested, whereby the question to answer defines, firstly, how the profit and loss and the capital invested are concretely defined and, secondly, whether the calculation refers to a historical or a future time period. As the original method of measuring returns, the MWR is a commonly used method, which is not only used in the context of asset management. The original formula for calculating a rate of return is as follows (formula 1):

Return $=\frac{\text { Profit or loss }}{\text { Capital invested }}$.
The above formula might suggest that the calculation of the return is a simple matter. However, this is not the case, as the calculation of the profit or loss as well as the invested capital can be performed in a wide variety of ways and based on different assumptions.

Based on the original formula for calculating the rate of return, various methods for calculating the MWR have been developed over time, whereby these solve the mathematical problem either exactly or using approximation methods. Examples of the former methods are the Internal Rate of Return, the Modified Internal Rate of Return, or the Adjusted Modified Internal Rate of Return and for the latter methods the Original Dietz Method or the Modified Dietz Method. ${ }^{1}$

## MWR AS THE ORIGINAL CONCEPT OF RETURN MEASUREMENT IN ASSET MANAGEMENT

It can be assumed that in asset management, the question of how to calculate a return has been discussed since the very beginning. Following the available literature and the current technical or scientific education, it seems that the beginning of the era of return measurement in asset management coincides with the publication of the book "Pension Funds: Measuring Investment Performance" by Peter O. Dietz in 1966. However, this appearance or presumption is highly questionable, since the question of measuring the rate of return cannot be considered as one that began in the 1960s. Asset management existed before 1960, so it can be assumed that this also applied to the question of measuring the rate of return. The fact that the development of the
methods of measuring the rate of return in asset management were strongly influenced by the explanations of Peter O. Dietz suggests that the publication of Peter O. Dietz's book, however, represents a milestone and the beginning of the era of modern return measurement in asset management.

## TWR AS THE MOST WIDELY USED CONCEPT FOR MEASUREMENT RETURNS IN ASSET MANAGEMENT

According to the original formula for calculating the return (see formula 1), the return is significantly determined by the invested capital. The invested capital corresponds to the money invested on average over the entire measurement period and depends to a considerable extent on the cash flows and their specific amounts as well as payment dates. Thus, the return is influenced not only by the decisions to buy and sell assets (management decisions), but also by the decisions to change the invested capital over time by means of cash flows (timing decisions). In asset management, these two types of decisions can be made by different decision makers, which is why a separation of the corresponding contributions to returns is required. This problem led, among others, to the establishment of the Time-Weighted Rate of Return method, which makes it possible to calculate a rate of return that is not influenced by the decisions about the invested capital or the timing decisions. Accordingly, the MWR can be decomposed into the return contribution of the management decisions and that of the timing decisions, and formula 1 can be rewritten accordingly (formula 2):

Return $=$ Return contribution of the management decisions

+ return contribution of the timing decisions
$=$ TWR + return contribution of the timing decisions.
Over time and with the increasing importance of external asset management, the TWR became established as market practice and best practice for calculating and presenting the return on assets. This was favored by the fact that the TWR enables a performance assessment that can be described as fair from the point of view of the asset managers, as well as a comparison with competitors and benchmarks. As the TWR became more widespread, criticism of the MWR and the underlying calculation assumptions increased, so that today the calculation of MWRs is rarely conducted or requested. Criticism of the MWR focused primarily on the implicit and unrealistic financing and reinvestment assumptions. In the technical and scientific discussion of the merits of TWRs and MWRs, this point of criticism was repeatedly raised, giving the impression that the calculation of the TWR would do without implicit financing and reinvestment assumptions.


## IMPLICIT FINANCING AND REINVESTMENT ASSUMPTIONS OF THE IRR

In the following, the explanations on the MWR focus on the Internal Rate of return (IRR), since the IRR is, on the one hand, one of the most widely used methods for calculating the MWR and, on the other hand, solves the mathematical problem exactly. For a given measurement period, the IRR causes the ending asset value discounted with the IRR to the beginning of the measurement period and the discounted interim cash flows to correspond exactly to the asset value at the beginning of the measurement period (formula 3):
$B M V_{0}=\frac{E M V_{T}}{(1+\operatorname{IRR})^{Y_{T}}}-\sum \frac{\mathrm{CI}_{\mathrm{t}}}{(1+\operatorname{IRR})^{\mathrm{Y}_{\mathrm{t}}}}+\sum \frac{\mathrm{CO}_{\mathrm{t}}}{(1+\mathrm{IRR})^{\mathrm{Y}_{\mathrm{t}}-0}}$
with
IRR = Internal rate of return (annualized) for a single period 0 to $T$,
$B M V_{0}=$ Asset value at the beginning of the period,
$E M V_{T}=$ Asset value at the end of the period,
$\mathrm{CI}_{\mathrm{t}}=$ Cash inflow at date t ,
$\mathrm{CO}_{\mathrm{t}}=$ Cash outflow at date t ,
$\mathrm{Y}_{\mathrm{T}} \quad=$ Length of measurement period (measured in years -365 days),
$\mathrm{Y}_{\mathrm{t}-0} \quad=$ Length of period between the beginning of the measurement period and the date of the cash flow (measured in years - 365 days).

The formula for calculating the IRR can also be rewritten as follows, so that the reinvestment of cash outflows and the financing of cash inflows assumed by the IRR concept become more obvious (formula 4):
$\mathrm{EMV}_{\mathrm{T}}=\mathrm{BMV}_{0} *(1+\mathrm{IRR})^{\mathrm{Y}_{\mathrm{T}}}+\sum \mathrm{CI}_{\mathrm{t}} *(1+\mathrm{IRR})^{\mathrm{Y}_{\mathrm{t}-\mathrm{T}}}-\sum \mathrm{CO}_{\mathrm{t}} *(1+\mathrm{IRR})^{\mathrm{Y}_{\mathrm{t}-\mathrm{T}}}$
with
$\mathrm{Y}_{\mathrm{t}-\mathrm{T}}=$ Length of period between the date of the cash flow and the end of the measurement period (measured in years - 365 days).

According to formula 4, the ending asset value is equal to the beginning asset value plus the sum of the cash inflows, the financing costs for the beginning asset value and the sum of the financing costs for the cash inflows, less the sum of the cash outflows and the sum of the reinvestment income for the cash outflows.

The implicit financing and reinvestment assumptions of IRR become clear by looking at the above formulas for calculating IRR and state that not only all cash inflows (incl. beginning asset value) and cash outflows, but also the financing costs for the cash inflows as well as the reinvestment returns on the cash outflows must be considered. The main criticism of the IRR concept here is not that the financing costs and reinvestment income are considered, but that these, expressed as a return, are identical and correspond to the IRR.

## IMPLICIT FINANCING AND REINVESTMENT ASSUMPTIONS OF THE TWR

The TWR concept states that the TWR is independent of the decisions about the capital invested (timing decisions). Based on the original formula for calculating returns and considering that there may be cash flows over time, the question is what the implicit financing and reinvestment assumption of the TWR is that causes the TWR to be independent of the decisions about the capital invested (timing decisions). The financing and reinvestment assumption of the TWR cause the returns of a portfolio with no cash flows and a portfolio with cash flows to be identical under otherwise identical conditions. It can be shown that this is exactly the case when the cash flows are compounded with the corresponding future TWRs to the end of the measurement period. As an example, two portfolios will be considered, where portfolio 1, in contrast to portfolio 2, has no intermediate cash flow. The ending asset values of the two portfolios are obtained from (formula 5 and 6):
$E M V_{P 1}=B M V_{P 1} *\left(1+T W R_{P 1,0-T}\right)^{Y_{T}}$
and
$E M V_{\mathrm{P} 2}=\mathrm{BMV}_{\mathrm{P} 2} *\left(1+\mathrm{TWR}_{\mathrm{P} 2,0-\mathrm{T}}\right)^{\mathrm{Y}_{\mathrm{T}}}+\mathrm{CI}_{\mathrm{P} 2} *\left(1+\text { Sub_TWR }_{\mathrm{P} 2, \mathrm{t}-\mathrm{T}}\right)^{\mathrm{Y}_{\mathrm{t}-\mathrm{T}}}$
with
$B M V_{P 2}=B M V_{P 1}$,
and
$\mathrm{TWR}_{\mathrm{P} 2,0-\mathrm{T}}=\mathrm{TWR}_{\mathrm{P} 1,0-\mathrm{T}}$,

| $E M V_{\text {P1 }}$ | $=$ | Asset value of portfolio 1 at the end of the period, |
| :---: | :---: | :---: |
| $B M V_{\text {P1 }}$ | $=$ | Asset value of portfolio 1 at the beginning of the period, |
| $\mathrm{TWR}_{\text {P1,0-T }}$ | = | Time-weighted rate of return (annualized) of portfolio 1 from the beginning to the end of the period, |
| $\mathrm{EMV}_{\text {P2 }}$ | $=$ | Asset value of portfolio 2 at the end of the period, |
| $\mathrm{BMV}_{\mathrm{P} 2}$ | = | Asset value of portfolio 2 at the beginning of the period, |
| $\mathrm{TWR}_{\text {P2,0-T }}$ | = | Time-weighted rate of return (annualized) of portfolio 2 from the beginning to the end of the period, |
| $\mathrm{CI}_{\text {P2 }}$ | = | Cash inflow of portfolio 2, and |
| Sub_TWR $\mathrm{P} 2, \mathrm{t}-\mathrm{T}$ | = | Sub-period time-weighted rate of return of portfolio 2 from time of the cash flow $t$ to the end of the period (annualized). |

After merging formula 5 and 6 and transforming, we get (formula 7):
$E M V_{\mathrm{P} 2}=E \mathrm{EMV}_{\mathrm{P} 1}+\mathrm{CI}_{\mathrm{P} 2} *\left(1+\text { Sub_TWR }_{\mathrm{P} 2, \mathrm{t}-\mathrm{T}}\right)^{\mathrm{Y}_{\mathrm{t}-\mathrm{T}}}$
The implicit financing and reinvestment assumption of the TWR is made clear by considering the above formulas and states that, just as with the IRR, all cash inflows (incl. beginning asset value) and cash outflows as well as the financing costs for the cash inflows and the reinvestment income on the cash outflows must be considered. The difference to the IRR, however, is that the financing costs and reinvestment income correspond to the specific TWR, which is the sub-period TWR valid from the time of the cash flow until the end of the measurement period. Accordingly, for the TWR, formula 4 can be rewritten as follows (formula 8):
$\mathrm{EMV}_{\mathrm{T}}=\mathrm{BMV}_{0} *\left(1+\mathrm{TWR}_{0-\mathrm{T}}\right)^{\mathrm{Y}_{\mathrm{T}}}+\sum \mathrm{CI}_{\mathrm{t}} *\left(1+\mathrm{Sub}_{-} \mathrm{TWR}_{\mathrm{t}-\mathrm{T}}\right)^{\mathrm{Y}_{\mathrm{t}-\mathrm{T}}}-\sum \mathrm{CO}_{\mathrm{t}} *\left(1+\mathrm{Sub}_{-} \mathrm{TWR}_{\mathrm{t}-\mathrm{T}}\right)^{\mathrm{Y}_{\mathrm{t}-\mathrm{T}}}$

## CONSIDERING EXPLICIT FINANCING COSTS AND REINVESTMENT INCOME

Following the concept of the Adjusted Modified Internal Rate of Return and taking into account explicit financing of cash inflows as well as explicit reinvestment of cash outflows, the cash flow stream of a portfolio can be converted into a cash flow stream without interim cash flows. ${ }^{2}$ This conversion has the advantage that the resulting cash flow stream in the case of a TWR calculation is independent of the actual cash flows. Accordingly, the TWR concept states that the intermediate cash flows must be included in the calculation in such a way that the return does not include any return contribution of the decisions about the invested capital (timing decisions), or that the TWR is independent of the amount and timing of the cash flows. The following four tables illustrate the above for four different cash flow streams.

Table 1: Cash flow stream without interim cash flows

| Date | Beginning market value | Cash flow | Annual TWR | Ending market value | Explicit reinvestment | Explicit borrowing |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 31.12 .2016 | 1'000.00 | 1'000.00 |  | 1'000.00 |  |  |
| 31.12 .2017 |  | - | -10.00\% | 900.00 | - | - |
| 31.12 .2018 |  | - | 5.00\% | 945.00 | - | - |
| 31.12.2019 |  | - | 15.00\% | 1'086.75 | - | - |
| Date | Cash flow stream without explicit reinvestment | Cash flow stream with explicit reinvestment IRR | Cash flow stream with explicit reinvestment future TWR |  |  |  |
| 31.12 .2016 | -1'000.00 | -1'000.00 | -1'000.00 |  |  |  |
| 31.12 .2017 | - | - | - |  |  |  |
| 31.12.2018 | - | - | - |  |  |  |
| 31.12.2019 | 1'086.75 | 1'086.75 | 1'086.75 |  |  |  |
|  |  |  |  |  |  |  |
| IRR (annu.) | 2.81\% | 2.81\% | 2.81\% |  |  |  |
| TWR (annu.) | 2.81\% |  |  |  |  |  |

Table 2: Cash flow stream with interim cash outflows

| Date | Beginning market value | Cash flow | Annual TWR | Ending market value | Explicit reinvestment | Explicit borrowing |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 31.12.2016 | 1'000.00 | 1'000.00 |  | 1'000.00 |  |  |
| 31.12.2017 |  | -100.00 | -10.00\% | 800.00 | 100.00 | - |
| 31.12.2018 |  | -200.00 | 5.00\% | 640.00 | 200.00 | - |
| 31.12.2019 |  | - | 15.00\% | 736.00 | - | - |
| Date | Cash flow stream without explicit reinvestment | Cash flow stream with explicit reinvestment IRR | Cash flow stream with explicit reinvestment future TWR |  |  |  |
| 31.12.2016 | -1'000.00 | -1'000.00 | -1'000.00 |  |  |  |
| 31.12.2017 | 100.00 | - | - |  |  |  |
| 31.12.2018 | 200.00 | - | - |  |  |  |
| 31.12.2019 | 736.00 | 1'041.47 | 1'086.75 |  |  |  |
|  |  |  |  |  |  |  |
| IRR (annu.) | 1.36\% | 1.36\% | 2.81\% |  |  |  |
| TWR (annu.) | 2.81\% |  |  |  |  |  |

Table 3: Cash flow stream with interim cash inflows

| Date | Beginning market value | Cash flow | Annual TWR | Ending market value | Explicit reinvestment | Explicit borrowing |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 31.12 .2016 | 1'000.00 | 1'000.00 |  | 1'000.00 |  |  |
| 31.12 .2017 |  | 100.00 | -10.00\% | 1'000.00 | - | 100.00 |
| 31.12.2018 |  | 200.00 | 5.00\% | 1'250.00 | - | 200.00 |
| 31.12.2019 |  | - | 15.00\% | 1'437.50 | - | - |
|  |  |  |  |  |  |  |
| Date | Cash flow stream without explicit reinvestment | Cash flow stream with explicit reinvestment IRR | Cash flow stream with explicit reinvestment future TWR |  |  |  |
| 31.12 .2016 | -1'000.00 | -1'000.00 | -1'000.00 |  |  |  |
| 31.12 .2017 | -100.00 | - | - |  |  |  |
| 31.12 .2018 | -200.00 | - | - |  |  |  |
| 31.12.2019 | 1'437.50 | 1'121.73 | 1'086.75 |  |  |  |
|  |  |  |  |  |  |  |
| IRR (annu.) | 3.90\% | 3.90\% | 2.81\% |  |  |  |
| TWR (annu.) | 2.81\% |  |  |  |  |  |

Table 4: Cash flow stream with interim cash inflows and outflows

| Date | Beginning market value | Cash flow | Annual TWR | Ending market value | Explicit reinvestment | Explicit borrowing |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 31.12.2016 | 1'000.00 | 1'000.00 |  | 1'000.00 |  |  |
| 31.12.2017 |  | -100.00 | -10.00\% | 800.00 | 100.00 | - |
| 31.12.2018 |  | 200.00 | 5.00\% | 1'040.00 | - | 200.00 |
| 31.12.2019 |  | - | 15.00\% | 1'196.00 | - | - |
| Date | Cash flow stream without explicit reinvestment | Cash flow stream with explicit reinvestment IRR | Cash flow stream with explicit reinvestment future TWR |  |  |  |
| 31.12 .2016 | -1'000.00 | -1'000.00 | -1'000.00 |  |  |  |
| 31.12 .2017 | 100.00 | - | - |  |  |  |
| 31.12.2018 | -200.00 | - | - |  |  |  |
| 31.12.2019 | 1'196.00 | 1'096.10 | 1'086.75 |  |  |  |
|  |  |  |  |  |  |  |
| IRR (annu.) | 3.11\% | 3.11\% | 2.81\% |  |  |  |
| TWR (annu.) | 2.81\% |  |  |  |  |  |

## CONCLUSION

This article addresses whether the MWR and TWR are two fundamentally different concepts. It shows that the TWR concept, like the IRR concept, contains implicit financing and reinvestment assumptions and that the TWR and IRR are ultimately "only" exceptional cases of a generic concept of return measurement. Moreover, it can be stated that the specific implicit financing and reinvestment assumptions in the case of the TWR concept mean that the TWR, unlike the IRR, is independent of the amount and timing of cash flows. This common market opinion has hereby been conceptualized and confirmed.

## CONTACT INFORMATION

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## ENDNOTES

1 For a discussion on different methods for calculating an internal rate of return, please see "Adjusted Modified Internal Rate of Return - Another Way to Calculate a Money Weighted Rate of Return" by Stefan Illmer published in the Journal of Performance Measurement, Fall 2021, Volume 26, number 1.

2 For a discussion on how to calculate an Adjusted Modified Internal Rate of Return, please see "Adjusted Modified Internal Rate of Return - Another Way to Calculate a Money Weighted Rate of Return" by Stefan Illmer published in the Journal of Performance Measurement, Fall 2021, Volume 26, number 1.

